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# Methods of Constructing Quantum Channels for Teleporting a General Two-qubit State

**Xiu-Lao Tian · Ming-Liang Hu · Xiao-Qiang Xi**

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**Abstract** By resorting to the tensor analysis, we derived an explicit CPM (channel parameter matrix) criterion based on the Bell state measurements. This criterion can be used to judge whether a four-qubit state can be employed as quantum channel or not for teleporting a general two-qubit state. According to this criterion, we presented a variety of quantum channels for faithful and deterministic teleportation of a general two-qubit state, which can provide more flexible choices for the experimenters.

**Keywords** Quantum teleportation · Bell state measurement

## 1 Introduction

Since the seminal work of Bennett et al. [1], quantum teleportation has attracted people's great attention both theoretically and experimentally [2–4] because of its novel features and potential applications in quantum information science. Substantial research effort has been devoted to the construction of various kinds of quantum channels, which enables the transmitting of an unknown quantum state from a sender, conventionally named Alice, to a receiver Bob at a distant location with the help of some classical information and without any particles physically transmitted. In the teleportation processing, there are two pivotal problems [5], the first is to construct the entangled channel, i.e., finding the proper parameters of the channel to realize the teleportation; the second is to choose the appropriate method of measurements. The Bell state measurement is often adopted because it is mature in the experiment.

The above two pivotal problems can be seen from the previous research about teleporting an unknown two-qubit state. Gorbachev and Trubilko [6] studied teleportation of an EPR pair with a three-qubit GHZ state. Shi et al. [7] and Cao et al. [8] considered probabilistic teleportation of a special two-qubit state using the three-qubit GHZ and W class

X.-L. Tian · M.-L. Hu (✉) · X.-Q. Xi
Department of Applied Mathematics and Applied Physics,
Xi'an Institute of Posts and Telecommunications, Xi'an 710061, China
e-mail: mingliang0301@xiyou.edu.cn

states, respectively. Later, Lee and Kim [9] reported teleportation of a two-qubit state via noisy quantum channel represented by the Werner state, and Lee [10] presented a setup for total teleportation of an entangled state. Then, Lee et al. [11] and Rigolin [12] investigated teleportation of an arbitrary two-qubit state, whereas Lee et al. did not explicitly construct a protocol, and Rigolin gave a protocol by defining 16 orthogonal $G$ states as the measurement bases. Subsequently, Dai et al. [13] presented a scheme for probabilistic teleportation of an arbitrary two-qubit state by employing two three-qubit W class states. In these references the special entangled channel are always considered.

The milestone work appears in [14], in which Yeo and Chua proposed an explicit protocol for faithfully teleporting an arbitrary two-qubit state by using the genuine four-qubit entangled states which are not reducible to a pair of Bell states. Later, Li et al. [15] also proposed a protocol to teleport an unknown two-qubit state by using a four-qubit entangled channel. However, a general method of choosing entangled channels for teleporting an arbitrary unknown two-qubit state is still an open problem.

In this paper, we will discuss this open problem by employing the tensor representation. We firstly derived a CPM (channel parameter matrix) criterion based on the Bell state measurements. This criterion can be utilized to determine whether a four-qubit state can serve as quantum channel or not for teleporting a general two-qubit state. Then based on this criterion, we propose a variety of quantum channels for faithfully teleporting a general two-qubit state. These protocols cover and complement the results published in [14, 15].

It must be emphasized that the term "general two-qubit state" we adopted in this paper should be interpreted as follows: if a four-qubit state passes our CPM criterion, then it can be employed to teleport any arbitrary two-qubit state. However, this statement is by no means implies that it covers all the four-qubit states that can teleport any type of two-qubit state, or in other words, there may be four-qubit states that do not pass this criterion but can still be used to teleport a "special" two-qubit state (as an example, the authors can resort to [10] of this paper).

## 2 The Tensor Analysis of Teleportation

Suppose the sender Alice has a general unknown two-qubit state $|\psi\rangle_{12}$ that needs to be teleported to the receiver Bob. With the help of tensor representation, $|\psi\rangle_{12}$ can be expressed as

$$|\psi\rangle_{12} = X^{ij}|ij\rangle \quad (i, j = 0, 1), \tag{1}$$

where $\{|ij\rangle\}$ is the orthonormal basis set, $X^{ij}$ is the arbitrary complex coefficient and satisfying the normalization condition $X^{ij}X^*_{ij} = 1$. Without loss of generality, we assume the quantum channel is composed of the following general four-qubit state

$$|\psi\rangle_{3456} = Y^{klmn}|klmn\rangle \quad (k, l, m, n = 0, 1), \tag{2}$$

where $Y^{klmn}Y^*_{klmn} = 1$, with particles (3, 4) belonging to Alice, and particles (5, 6) belonging to Bob. Thus the joint system composed of the state to be teleported and the quantum channel can be expressed as

$$|\psi\rangle = |\psi\rangle_{12} \otimes |\psi\rangle_{3456} = X^{ij}Y^{klmn}|ijklmn\rangle. \tag{3}$$

To perform the joint measurements, one can choose the four maximally entangled Bell states as a complete set on which the total state can be decomposed. The four Bell states are given by

$$|\varphi^{1,2}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \qquad |\varphi^{3,4}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle). \tag{4}$$

With the aid of the Bell basis, the state of the joint system can be decomposed as follows

$$|\psi\rangle = X^{ij} Y^{klmn} T^{\alpha}_{ik} T^{\beta}_{jl} |\alpha\beta\rangle \otimes |mn\rangle = X^{ij} Y^{klmn} T^{\alpha\beta}_{ikjl} |\alpha\beta mn\rangle, \tag{5}$$

where the decomposition is executed for particle-pairs $(1,3)$ and $(2,4)$, with $|\alpha\rangle, |\beta\rangle = |\varphi^s\rangle$ $(s = 1,2,3,4)$. $T^{\alpha}_{ik}$ and $T^{\beta}_{jl}$ are basis transformation operators which are defined by their actions on the Bell states $T^{\alpha}_{ik}|\alpha\rangle = |ik\rangle$, $T^{\beta}_{jl}|\beta\rangle = |jl\rangle$.

If one define the decomposition operator [16, 17] as

$$\sigma^{\alpha\beta mn}_{ij} = 4 Y^{klmn} T^{\alpha\beta}_{ikjl}, \tag{6}$$

then the state of the joint system can be rewritten as

$$|\psi\rangle = \frac{1}{4} X^{ij} \sigma^{\alpha\beta mn}_{ij} |\alpha\beta mn\rangle. \tag{7}$$

In order to realize the teleportation, Alice first performs two joint Bell state measurements on particle-pairs (1, 3) and (2, 4) belong to her. Then the state of particles (5, 6) will be collapsed into one of the following sixteen states with equal probabilities

$$|\psi\rangle^{(\alpha\beta)} = X^{ij} \sigma^{(\alpha\beta)mn}_{ij} |mn\rangle, \tag{8}$$

where $(\alpha\beta)$ denotes the free index.

From (8) one can conclude that if $\sigma^{(\alpha\beta)}$ is a unitary operator, Alice can inform Bob her measurement results via a classical channel, then Bob can recover the unknown state on particles (5, 6) faithfully by performing unitary operation $\sigma^{(\alpha\beta)\dagger}$ conditioned on Alice's outcome. If $\sigma^{(\alpha\beta)}$ is invertible but not unitary, however, Bob can only recover the two-qubit state probabilistically by applying certain appropriate unitary operations. If $\sigma^{(\alpha\beta)}$ is not invertible, Bob cannot recover the two-qubit state.

## 3 Constructing the Quantum Channel for Teleportation

Although the decomposition operator $\sigma^{(\alpha\beta)}$ can tell us if a channel is suitable for teleportation or not, the experimenters prefer to know how to construct a quantum channel for teleportation. So we will further discuss the relation between decomposition operators and the quantum channel, and aim at revealing a more convenient and feasible criterion. To sketch our central idea, let us choose the two orthonormal basis sets of the four-dimensional Hilbert space as $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ and $\{|\varphi^1\rangle, |\varphi^2\rangle, |\varphi^3\rangle, |\varphi^4\rangle\}$, then it is straightforward to obtain the basis transformation matrix as

$$T = \begin{pmatrix} T^1_{00} & T^2_{00} & T^3_{00} & T^4_{00} \\ T^1_{01} & T^2_{01} & T^3_{01} & T^4_{01} \\ T^1_{10} & T^2_{10} & T^3_{10} & T^4_{10} \\ T^1_{11} & T^2_{11} & T^3_{11} & T^4_{11} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & -1 & 0 & 0 \end{pmatrix}, \tag{9}$$

where the superscripts 1, 2, 3, and 4 in the entries of the matrix correspond to the four Bell states $|\varphi^1\rangle$, $|\varphi^2\rangle$, $|\varphi^3\rangle$ and $|\varphi^4\rangle$.

After obtaining the basis transformation matrix $T$, the main problem now is to find out the optimal channel parameters for perfect teleportation of the general two-qubit state. To this end, we write the decomposition operator in the following matrix form

$$\sigma^{(\alpha\beta)} = \begin{pmatrix} \sigma_{00}^{(\alpha\beta)00} & \sigma_{01}^{(\alpha\beta)00} & \sigma_{10}^{(\alpha\beta)00} & \sigma_{11}^{(\alpha\beta)00} \\ \sigma_{00}^{(\alpha\beta)01} & \sigma_{01}^{(\alpha\beta)01} & \sigma_{10}^{(\alpha\beta)01} & \sigma_{11}^{(\alpha\beta)01} \\ \sigma_{00}^{(\alpha\beta)10} & \sigma_{01}^{(\alpha\beta)10} & \sigma_{10}^{(\alpha\beta)10} & \sigma_{11}^{(\alpha\beta)10} \\ \sigma_{00}^{(\alpha\beta)11} & \sigma_{01}^{(\alpha\beta)11} & \sigma_{10}^{(\alpha\beta)11} & \sigma_{11}^{(\alpha\beta)11} \end{pmatrix}. \tag{10}$$

Then from (6), one can easily obtain the channel parameter matrix (CPM) and the basis transformation matrix for subsystem composed of qubits $\{1, 2, 3, 4\}$ as

$$Y = \begin{pmatrix} Y^{0000} & Y^{0100} & Y^{1000} & Y^{1100} \\ Y^{0001} & Y^{0101} & Y^{1001} & Y^{1101} \\ Y^{0010} & Y^{0110} & Y^{1010} & Y^{1110} \\ Y^{0011} & Y^{0111} & Y^{1011} & Y^{1111} \end{pmatrix}, \tag{11}$$

$$T^{(\alpha\beta)} = \begin{pmatrix} T_{00}^{(\alpha)} & T_{10}^{(\alpha)} \\ T_{01}^{(\alpha)} & T_{11}^{(\alpha)} \end{pmatrix} \otimes \begin{pmatrix} T_{00}^{(\beta)} & T_{10}^{(\beta)} \\ T_{01}^{(\beta)} & T_{11}^{(\beta)} \end{pmatrix} = \begin{pmatrix} T_{0000}^{(\alpha\beta)} & T_{0010}^{(\alpha\beta)} & T_{1000}^{(\alpha\beta)} & T_{1010}^{(\alpha\beta)} \\ T_{0001}^{(\alpha\beta)} & T_{0011}^{(\alpha\beta)} & T_{1001}^{(\alpha\beta)} & T_{1011}^{(\alpha\beta)} \\ T_{0100}^{(\alpha\beta)} & T_{0110}^{(\alpha\beta)} & T_{1100}^{(\alpha\beta)} & T_{1110}^{(\alpha\beta)} \\ T_{0101}^{(\alpha\beta)} & T_{0111}^{(\alpha\beta)} & T_{1101}^{(\alpha\beta)} & T_{1111}^{(\alpha\beta)} \end{pmatrix}. \tag{12}$$

From (9) one can see that the four basis transformation matrices are given by

$$T^{(1)} = \frac{1}{\sqrt{2}} I, \qquad T^{(2)} = \frac{1}{\sqrt{2}} \sigma_z, \qquad T^{(3)} = \frac{1}{\sqrt{2}} \sigma_x, \qquad T^{(4)} = -\frac{1}{\sqrt{2}} i\sigma_y, \tag{13}$$

where $\sigma_i$ $(i = x, y, z)$ are the usual Pauli matrices and $I$ is the identity matrix. Equations (12) and (13) imply that $2T^{(\alpha\beta)}$ is always a unitary operator, combining of this with (6) one can conclude that if $2Y$ is a unitary operator, then Bob can recover the teleported state on particles (5, 6) by performing corresponding unitary operations conditioned on Alice's measurement results. If $2Y$ is invertible but not unitary, Bob can only recover the teleported state probabilistically by applying relevant unitary operations. If $2Y$ is not invertible, Bob can never recover the unknown state.

From another point of view, if $2Y$ is a unitary operator, then the norm of the determinant of it equals to unity, i.e., $|\det(2Y)| = 1$, however, it is invertible but not unitary if $|\det(2Y)| \neq 0, 1$ and not invertible if $|\det(2Y)| = 0$. Thus from the determinant of the CPM one can easily determine whether a four-qubit state can be employed as quantum channel or not for teleporting a general two-qubit state. In fact, the condition $|\det(2Y)| = 1$ (together with the normalization condition $Y^{klmn} Y^*_{klmn} = 1$ of the quantum channel) is a necessary and sufficient condition for deterministic and faithful teleportation of a general two-qubit state (a strict proof can be found in the Appendix). For convenience of representation, we will call this criterion the CPM criterion in the following discussion.

To construct the quantum channel for faithfully and deterministically teleporting a completely unknown two-qubit state, the operator $2Y$ should obeys our CPM criterion exactly. So far as we know, there are several schemes reported to teleport the general two-qubit state using four-qubit quantum channel [12, 14, 15]. For example, the following two genuine four-qubit entangled states $|\Pi^{\pm}\rangle = (|0000\rangle \pm |0011\rangle - |0101\rangle + |0110\rangle + |1001\rangle +$

$|1010\rangle \mp |1100\rangle + |1111\rangle)/2\sqrt{2}$ proposed by Yeo and Chua [14]. Using (11) they can be represented in the form of the channel parameter matrices as

$$Y^{\Pi} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & \mp 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \pm 1 & 0 & 0 & 1 \end{pmatrix}. \tag{14}$$

It can be easily checked that $|\det(2Y^{\Pi})| = 1$, thus the teleportation can be perfectly achieved. The other quantum channels proposed in the literature can also be checked in a similar way, and the results show that they all satisfy our CPM criterion, i.e., $|\det(2Y)| = 1$ and $2Y$ is a unitary operator.

Here we propose a variety of different quantum channels for faithful teleportation of a general two-qubit state. As pointed above, to ensure the success of faithful teleportation, the CPM should be such constructed so that $|\det(2Y)| = 1$. One way to guarantee this is to arrange $Y = P/2$ in the form of a permutation matrix $P$ as follows

$$P = \begin{pmatrix} P^{0000} & P^{0100} & P^{1000} & P^{1100} \\ P^{0001} & P^{0101} & P^{1001} & P^{1101} \\ P^{0010} & P^{0110} & P^{1010} & P^{1110} \\ P^{0011} & P^{0111} & P^{1011} & P^{1111} \end{pmatrix}, \tag{15}$$

where there has exactly one entry 1 in each row and each column and 0's elsewhere. This matrix satisfies the relations $P^T = P^{-1}$ and $\det P = \pm 1$, which clearly meets our CPM criterion, and can be employed to construct 24 different quantum channels for faithful teleportation of any two-qubit states. Moreover, one can show that a generalization of (15) with one entry $e^{-i\theta_k}$ ($\theta_k$is an arbitrary phase angle) in each row and each column and 0's elsewhere can also fulfill the CPM criterion, from which one can construct different quantum channels for the success of faithfully teleporting an unknown two-qubit state.

In [14], the authors proposed a protocol for faithfully teleporting a general two-qubit state via a four-qubit state which is not reducible to a pair of Bell states, with the CPM given by

$$Y = \frac{1}{2} \begin{pmatrix} \cos\theta & 0 & 0 & \sin\theta \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ -\sin\theta & 0 & 0 & \cos\theta \end{pmatrix}. \tag{16}$$

It is clear that $Y$ expressed in (16) fulfills our CPM criterion exactly, thus can be employed to faithfully teleport a general two-qubit state. Here we show that a slight change of (16) can also be used to construct quantum channels for faithfully teleporting a general two-qubit state. We know that the norm of the determinant of a matrix is invariant when exchanging rows or columns, so is the unitarity of it. Thus one can construct quantum channels by exchanging rows or columns of the matrix expressed in (16). In fact, if one multiplies the $m$-th row or column of the matrix with an arbitrary phase factor $e^{-i\theta}$, the unitarity of it is still unchanged, which ensures successful construction of different quantum channels for teleporting a general unknown two-qubit state.

In the following we propose another scheme for faithful teleportation of the two-qubit state based on the CPM criterion. We choose $2Y$ as a product of two general unitary matrices

$$2Y = U_1 \otimes U_2, \tag{17}$$

where the two unitary matrices are given by

$$U_{1,2}(\zeta\varsigma\eta) = \begin{pmatrix} e^{-i\zeta_{1,2}}\cos\eta_{1,2} & -e^{-i\varsigma_{1,2}}\sin\eta_{1,2} \\ e^{i\varsigma_{1,2}}\sin\eta_{1,2} & e^{i\zeta_{1,2}}\cos\eta_{1,2} \end{pmatrix} \quad (0 \le \zeta, \varsigma \le 2\pi, 0 \le \eta \le \pi/2). \tag{18}$$

Clearly, (17) obeys our CPM criterion exactly, with the decomposition operator given by

$$\sigma^{(\alpha\beta)} = 2(U_1 \otimes U_2)(T^{(\alpha)} \otimes T^{(\beta)}) = 2(U_1 T^{(\alpha)}) \otimes (U_2 T^{(\beta)}). \tag{19}$$

One can construct different quantum channels by choosing different parameters $\zeta_{1,2}$, $\varsigma_{1,2}$ and $\eta_{1,2}$, or by exchanging rows and columns of the unitary matrix of (17). Bob can reconstruct the unknown two-qubit state by performing relevant unitary operation $\sigma^{(\alpha\beta)\dagger}$ conditioned on the four bits classical information received from Alice, with successful probabilities and fidelities both reach unity. If fact, the quantum channel expressed in (17) can be reduced to a pair of two-qubit entangled states

$$|\psi\rangle_{3456} = Y_1^{kl}|kl\rangle_{35} \otimes Y_2^{mn}|mn\rangle_{46}, \tag{20}$$

where

$$Y_{1,2} = \begin{pmatrix} Y_{1,2}^{00} & Y_{1,2}^{10} \\ Y_{1,2}^{01} & Y_{1,2}^{11} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\zeta_{1,2}}\cos\eta_{1,2} & -e^{-i\varsigma_{1,2}}\sin\eta_{1,2} \\ e^{i\varsigma_{1,2}}\sin\eta_{1,2} & e^{i\zeta_{1,2}}\cos\eta_{1,2} \end{pmatrix}. \tag{21}$$

Clearly, this is a two-qubit entanglement channel [11] since both the states $|\psi\rangle_{35} = Y_1^{kl}|kl\rangle_{35}$ and $|\psi\rangle_{46} = Y_2^{mn}|mn\rangle_{46}$ are maximally entangled with the concurrence [18, 19] equal to one.

## 4 Conclusion

In summary, we have derived an explicit CPM criterion which can be used to determine which type of the four-qubit states can be used as quantum channels for teleporting a general two-qubit state. The derivation of this criterion is based on and thus is applicable for Bell state measurements. According to this CPM criterion, we presented a variety of quantum channels for faithfully and deterministically teleporting a general two-qubit unknown state. In fact, as long as $2Y$ is a natural representation of the $SU(4)$ group, perfect teleportation can always be realized with the Bell state measurement.

The CPM criterion and the quantum channels proposed in the present work is the simplest but physically fundamental and unavoidable one. Nevertheless, it would be of more importance to extend this analysis to the problem of teleporting a general $n$-qubit state ($n > 2$). In fact, our further studies show that there exists a similar condition on $2n$-qubit entanglement for teleporting an $n$-qubit state, which possess more intriguing and novel properties. Particularly, from the point of group theory, one can find flexible quantum channels for perfect teleportation. We will discuss this problem in future works.

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## Appendix: Proof of the CPM Criterion

In Sect. 3 we mentioned that the condition $|\det(2Y)| = 1$ (together with the normalization condition $Y^{klmn}Y^*_{klmn} = 1$ of the quantum channel) is in fact a necessary and sufficient condition for deterministic and faithful teleportation of a general two-qubit state, i.e., if $|\det(2Y)| = 1$ and $Y^{klmn}Y^*_{klmn} = 1$, then $2Y$ is a unitary operator, and vice versa. In this appendix we gave a proof of this criterion.

From the Hadamard theorem, we know that if $A$ is a $n \times n$ positive semidefinite matrix with elements $a_{ij}$, then $\det A \le \prod_{i=1}^{n} a_{ii}$, and the equality holds if and only if $A$ is a diagonal matrix. From this theorem, we immediately arrive at the following two Propositions.

**Proposition 1** *Let $A$ be an $n \times n$ matrix with elements $a_{ij}$, then*

$$|\det A| \le \prod_{i=1}^{n}\left(\sum_{j=1}^{n}|a_{ij}|^2\right)^{\frac{1}{2}}. \tag{A.1}$$

*Equality holds if and only if the row vectors of $A$ orthogonal with each other.*

*Proof* Since $A$ is an $n \times n$ matrix, $B = AA^\dagger$ is clearly a positive semidefinite matrix, thus from the Hadamard theorem one obtain

$$\det AA^\dagger = \det B \le \prod_{i=1}^{n} b_{ii} = \prod_{i=1}^{n}\sum_{j=1}^{m}|a_{ij}|^2. \tag{A.2}$$

Moreover, for arbitrary matrix $A$ one has the equality $\det A \cdot \det A^\dagger = |\det A|^2$. Thus by taking the square roots of both sides of (A.2), one can immediately arrive at (A.1). □

**Proposition 2** *For arbitrary positive numbers $x_1, x_2, \ldots, x_n$, one has the following inequality*

$$x_1 + x_2 + \cdots + x_n \ge n(x_1 \cdot x_2 \cdots x_n)^{1/n}. \tag{A.3}$$

*Equality holds if and only if $x_1 = x_2 = \cdots = x_n$.*

From the above two propositions, we can arrive at the following theorem.

**Theorem** *Let $A$ be an $n \times n$ matrix with elements $a_{ij}$, then the necessary and sufficient condition for $|\det A| = 1$, $\mathrm{tr}(AA^\dagger) = n$ is $AA^\dagger = I$, i.e., $A$ is a unitary matrix.*

*Proof* Noted that the equality $\mathrm{tr}(AA^\dagger) = n$ is equivalent to $\sum_{i=1}^{n}\sum_{j=1}^{n}|a_{ij}|^2 = n$, thus from the equality $AA^\dagger = I$, one can easily demonstrate $|\det A| = 1, \mathrm{tr}(AA^\dagger) = n$. On the other hand, for $|\det A| = 1, \mathrm{tr}(AA^\dagger) = n$, from Propositions 1 and 2 one can obtain

$$|\det A|^2 \le \prod_{i=1}^{n}\sum_{j=1}^{n}|a_{ij}|^2 \le \left(\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}|a_{ij}|^2\right)^n = 1. \tag{A.4}$$

Since $|\det A| = 1$, again, from Propositions 1 and 2, one has $\sum_{j=1}^{n}|a_{ij}|^2 = 1$, and the row vectors of $A$ orthogonal with each other, thus the equality $AA^\dagger = I$ holds. □

When $A = 2Y$ with $Y$ being the four-qubit channel parameter matrix which satisfies the normalization condition $Y^{klmn}Y^{*}_{klmn} = 1$ (or equivalently, $\sum_{i=1}^{4}\sum_{j=1}^{4}|a_{ij}|^2 = 4$ or $\mathrm{tr}(AA^\dagger) = 4$), the theorem proved here reduces to that presented in Sect. 3 of this work. Moreover, we would like to point out that the theorem proved here will be very helpful when extending the analysis to the problem of teleporting a general $n$-qubit state ($n > 2$).

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